MAA 10 Koehajoitustehtävien ratkaisut

1. a) $\int 3x^2 + 5x^7 \, dx = x^3 + \frac{5}{8}x^8 + C$

b) $\int \sin x + \cos 2x \, dx = -\cos x + \frac{2}{2}\int \cos 2x \, dx$

$= -\cos x + \frac{1}{2}\int 2\cos 2x \, dx$

$= -\cos x + \frac{1}{2}\sin 2x + C$

c) $\int_0^5 |2x-6| \, dx$

tarkistetaan itseisarvo

$2x - 6 < 0 \iff x < 3$

$\Rightarrow \int_0^5 |2x-6| \, dx = \int_0^3 6-2x \, dx + \int_3^5 2x-6 \, dx$

$= \Big/_0^3 6x - x^2 + \Big/_3^5 x^2 - 6x$

$= [6\cdot 3 - 3^2 - (6\cdot 0 - 0^2)] + [(5^2 - 6\cdot 5) - (3^2 - 6\cdot 3)]$

$= [18-9] + [(25-30) - (9-18)]$

$= 9 - 5 + 9 = 13$

d) $\int_2^3 \frac{dx}{1-x} = \int_2^3 (1-x)^{-1} dx = -\int_2^3 (1-x)^{-1} dx$

$= -\Big/_2^3 \ln(|1-x|) \, dx = -\ln(|1-3|) - (-\ln|1-2|)$

$= -\ln|-2| + \ln(|-1|)$

$= -\ln 2$

MAA 10 Harjoitusteht. ratkaisut 2.

1. e) $\int x\sqrt{x^2+3}\,dx = \frac{2}{2}\int x\,(x^2+3)^{\frac{1}{2}}dx$

$= \frac{1}{2}\int 2x\,(x^2+3)^{\frac{1}{2}}\,dx = \frac{1}{2}\left[\frac{2}{3}\cdot(x^2+3)^{\frac{3}{2}}\right] + C$

$= \frac{1}{3}\sqrt{(x^2+3)^3} + C$

2. Ratkaistaan leikkauspisteet

$\begin{cases} y^2 = 4x \\ 4x - 3y = 4 \end{cases}$ Sij. (1) yhtälöön (2).

$y^2 - 3y - 4 = 0$

$y = \frac{3 \pm \sqrt{(-3)^2 - 4\cdot 1\cdot(-4)}}{2\cdot 1} = \frac{3 \pm 5}{2} \Rightarrow y = \begin{cases} 4 \\ -1 \end{cases}$

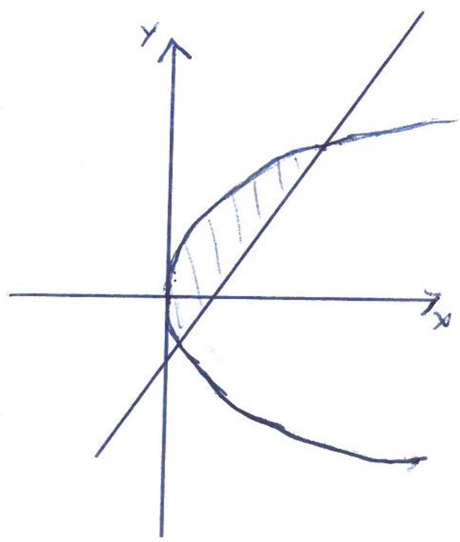
kun $y = 4 \Rightarrow x = \frac{4^2}{4} = 4 \Rightarrow$ piste $(4, 4)$

kun $y = -1 \Rightarrow x = \frac{(-1)^2}{4} = \frac{1}{4} \Rightarrow$ piste $(\frac{1}{4}, -1)$

Piirtoa varten ratkaistaan suora

$4x - 3y = 4 \Leftrightarrow$

$y = \frac{4}{3}x - \frac{4}{3}$



Suoritetaan integrointi y:n suhteen

Suora: $x = \frac{3}{4}y + 1 = f(y)$

Paraabeli $x = \frac{1}{4}y^2 \quad = g(y)$

Välillä $[-1, 4]$ $f(y) \geq g(y)$

$\Rightarrow A = \int_{-1}^{4} f(y) - g(y)\,dy = \int_{-1}^{4} \frac{3}{4}y + 1 - \frac{1}{4}y^2\,dy$

$= \Big/_{-1}^{4} \frac{3}{2\cdot 4}y^2 + y - \frac{1}{3\cdot 4}y^3 = \Big/_{-1}^{4} \frac{3}{8}y^2 + y - \frac{1}{12}y^3$

$= \frac{125}{24} \approx 5{,}20832 \approx 5{,}21$

MAA 10 Koeharjoitus ratkaisut 3.

3. $f(x) = \sqrt{2x+4}$

a) x-akselin leikkaus

$\sqrt{2x+4} = 0 \Leftrightarrow 2x+4=0 \Leftrightarrow x=-2$

$\Rightarrow (-2,0)$

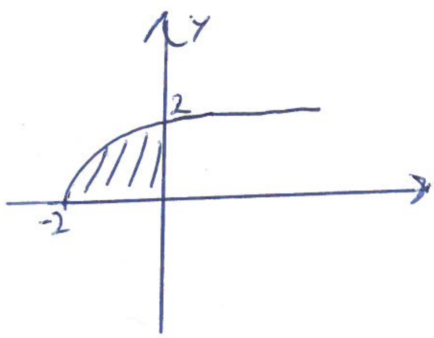
y-akseli, kun $x=0$

$f(0) = \sqrt{0+4} = 2 \qquad \Rightarrow (0,2)$

b) Integroidaan leikkauspisteestä toiseen

$$\int_{-2}^{0} \sqrt{2x+4}\,dx = \frac{1}{2}\int_{-2}^{0} 2(2x+4)^{\frac{1}{2}}\,dx$$

$$= \frac{1}{2} \Big/_{-2}^{0} \frac{2}{3}(2x+4)^{\frac{3}{2}} = \frac{1}{3}\left[4^{3/2} - (-4+4)^{3/2}\right]$$



$$= \frac{\sqrt{4^3}}{3} = 2,666\ldots$$

c) Ratkaistaan x

$y = \sqrt{2x+4} \Leftrightarrow y^2 = 2x+4 \Leftrightarrow x = \frac{1}{2}y^2 - 2$

$$V = \pi\int_{0}^{2}\left(\frac{1}{2}y^2-2\right)^2 dx = \frac{64\pi}{15} \quad \text{(laskimella)}$$